Math 55 Discussion problems 14 Feb

1. Find the decimal expansion of the number with the n-digit base seven expansion $(111...111)_7$ (with n 1's). [Hint: Use the formula for the sum of the terms of a geometric progression.]
2. How many zeros are at the end of the binary expansion of $(100_{10})!$?
3. Determine whether the integers in each of these sets are pairwise relatively prime.

   (a) $21, 34, 55$

   (b) $14, 17, 85$

   (c) $25, 41, 49, 64$

   (d) $17, 18, 19, 23$

4. Use the Euclidean algorithm to find

   (a) gcd(1, 5)

   (b) gcd(100, 101)

   (c) gcd(123, 277)

   (d) gcd(1529, 14039)

   (e) gcd(1529, 14038)

   (f) gcd(11111, 111111)

5. Prove that for every positive integer n, there are n consecutive composite integers. [Hint: Consider the n consecutive integers starting with $(n+1)!+2$.]
6. Show that if a and b are both positive integers, then $(2^a - 1)$ **mod** $(2^b - 1) = 2^{a \textbf{ mod } b} - 1$.
7. Use the question above to show that if a and b are positive integers, then $\gcd(2^a - 1, 2^b - 1) = 2^{\gcd(a,b)} - 1$.